DETERMINATION OF THE PARAMETERS OF RELAXATION KERNELS FROM THE RESULTS OF A WAVE EXPERIMENT

N. M. Ivanov and V. P. Muzychenko

UDC 539.3.5:678

It is necessary to perform the appropriate experiments over a wide range of variation of the time for a complete description of the mechanical behavior of polymeric materials possessing pronounced viscoelastic properties. The reaction of the material to loads whose variation in time can encompass the range from one period per hour to tens of Megahertz and up to several months and even years for creep [1] has been discussed in similar experiments. Many papers devoted to investigations of the mechanical properties of linear viscoelastic media have been published recently. An extensive bibliography in which the procedure and results of these investigations are described is given in [2-7]. It is necessary for the solution of boundary-value problems of viscoelasticity associated with the propagation of nonsteady waves to develop theoretical–experimental methods of constructing viscoelastic models corresponding to conditions which most closely approach in duration and intensity the loads which trigger transitional processes. It is proposed to use the method of numerical solution of the inverse problem for a nonstationary rod loaded by a short pulse of longitudinal strain together with the traditional experimental dynamic methods of determining the parameters of viscoelastic models [8-13]. The duration of the input signal as well as the coordinates of characteristic points on oscillograms which have recorded a transitional process in a specific cross section of a polymeric sample rod encompassing several reflected (for example, up to ten) waves preferably not interacting with each other can serve as the initial data for such a problem. The proposed method is an important modification of the traveling-wave method [1] but is most similar in its experimental implementation to the "impact wave" method (IWM) applied [14] in the radiography of structural elements. The appropriate upper limit to the frequencies of the method lies above 10-20 kHz, the maximum possible for the free vibration method or the resonance method [11-13] bounded by the highest harmonic, which can be excited in a sample with the help of the smallest additional mass due to excitation of a "standing" wave [1], and lower than the ultrasonic sounding methods, of which the extremely low levels of the acting load intensities, which do not correspond to the actual ones, are characteristic. In contrast to the resonance method, the proposed method is freed of the presence of an electromechanical moment of the forces from interaction of the "exciting" coil with a magnetic field, which leads to large errors [15]. The method has been described in [16] in the formation outlined below, but it was oriented only to finding the parameters of the Rabotnov kernels [17] from only a single signal, which reduced the accuracy of the method.

The graphical time dependence of the longitudinal strain recorded by a measuring system for a freely suspended rod made out of polymethyl methacralate with a length of 0.985 m and a thickness of 0.0095 m when bullets from a pneumatic rifler are shot into the end is given in Fig. 1.

It is proposed that the dependence of the longitudinal displacements u, strains ε, and stresses σ on the time t and the Lagrangian coordinate x is described with zero boundary conditions by the system of equations:

$$\varepsilon = \partial u/\partial x, \; \partial\sigma/\partial x = \rho\partial^2 u/\partial t^2; \tag{1}$$

$$\frac{\sigma}{E_0} = \varepsilon - \int_0^t T(t-\tau)\,\varepsilon(x, \tau)\,d\tau, \tag{2}$$

where ρ is the density of the material; E_0, instantaneous modulus of elasticity; and T(t), a function of the stress relaxation rate. In specifying the boundary conditions we shall make use of the following approximation:

$$\varepsilon(L, t) = 0; \tag{3}$$

Daugavpils. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 121-127, January-February, 1983. Original article submitted December 15, 1981.

 0021-8944/83/2401-0106$07.50

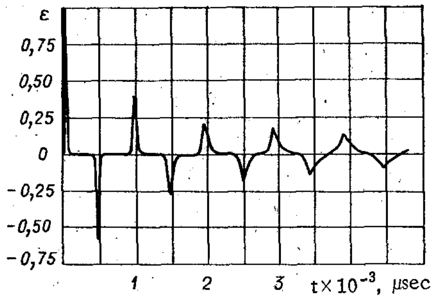

Fig. 1

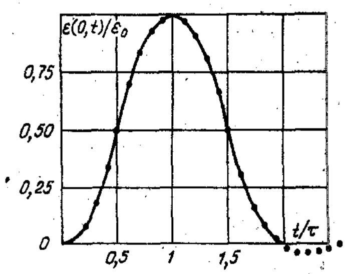

Fig. 2

$$\frac{\varepsilon(0,t)}{\varepsilon_0}=\begin{cases}2(t/\tau)^2 & \text{for } 0\leqslant t/\tau<1/2,\\ 1-2(t/\tau-1)^2 & \text{for } \frac{1}{2}\leqslant t/\tau\leqslant 3/2,\\ 2(t/\tau-2)^2 & \text{for } \frac{3}{2}\leqslant t/\tau\leqslant 2,\\ 0 & \text{for } t/\tau>0,\end{cases} \tag{5}$$

where 2τ is the duration of the input pulse (Fig. 2, solid curve) and ε_0 is the maximum value of the strain.

Applying the operational method, one can represent the solution of the problem (1)-(4) similarly to [18] in the form of a superposition of direct and reverse waves. We obtain the Laplace transformation for the strain in cross section x in the form

$$\bar{\varepsilon}(x,p)=\sum_{k=0}^{l}\left[\overline{W}(t_{k1},p)\,e^{-t_{k1}p}-\overline{W}(t_{k2},p)\,e^{-t_{k2}p}\right], \tag{5}$$

where l is the number of reflections of the wave from the free end of the rod being discussed;

$$\overline{W}(t_k,p)=\bar{\varepsilon}(0,p)\,e^{-\frac{t_k p\overline{T}(p)}{1-\overline{T}(p)+\sqrt{1-\overline{T}(p)}}}; \tag{6}$$

$$\frac{\bar{\varepsilon}(0,p)}{\varepsilon_0}=\frac{4}{p^3\tau^2}\left(1-e^{-p\tau}\right)\left(1-e^{-\frac{p\tau}{2}}\right), \tag{7}$$

$$t_{k1}=\frac{2Lk+x}{C_0},\quad t_{k2}=\frac{2L(k-1)-x}{C_0},\quad C_0=\sqrt{\frac{E_0}{\rho}}.$$

The problem simplifies significantly if the duration of the input pulse is appreciably less than the time interval L/C_0 during which the wave traverses the length of the rod. In this case restricting ourselves to a discussion only of direct waves recorded in the upper half plane of Fig. 1, we obtain instead of (5)

$$\bar{\varepsilon}_1(x,p)=\sum_{k=1}^{l}\overline{W}\left(\frac{2Lk}{C_0},\ p\right)e^{-\frac{2Lk}{C_0}p},$$

figuring the time from the instant x/C_0. We shall assume the first pulse recorded by the measuring system in cross section x to be the input signal. Using the well-known retardation theorem, we find

$$\varepsilon_1(x,t)=\sum_{k=1}^{n}W(t_k,t-t_k)\,h(t-t_k), \tag{8}$$

where $t_k = 2Lk/C_0$ and h(t) is the Heaviside unit function.

Thus, calculation of the strains in a model problem reduces to inversion of the image (6). It is proposed for the solution of this problem to use an economic algorithm constructed with the help of an approximation of the original f(t) by a trigonometric system [19]:

$$f(t)\approx f_n(t)=\sum_{m=1}^{n}C_m\sin\left[(2m-1)\arccos e^{-\sigma t}\right], \tag{9}$$

where $\sigma > 0$ is an arbitrary parameter. The coefficients C_m are determined from the linear system

$$\sum_{i=1}^{m}a_{mi}C_i=\frac{4^m}{\pi}\sigma\bar{f}(p_m),\quad m=1,2,\ldots,n, \tag{10}$$

where $p_m = (2m-1)\sigma$ and $\bar{f}(p)$ is the specified image. The elements of the triangular matrix $\|a_{mi}\|$ are constant quantities; their values are given, for example, in [19].

One can find the solution of the system (10) by taking account of the fact that $a_{mn} = 1$ with the help of the recurrence relationships:

$$C_1 = \frac{4}{\pi}\sigma\bar{f}(p), \quad C_m = \frac{4^m}{\pi}\sigma\bar{f}(p_m) - \sum_{i=1}^{m} a_{mi}C_i, \quad m = 2, 3, \ldots, n.$$

It is necessary for the computer calculation of a single value of $\bar{f}_n$ to perform $(n^2 + 3n)/2 + 1$ multiplication operations, $n^2 - n + 1$ addition–subtraction operations, and n calculations of the functions $\bar{f}(p_m)$ and $\sin[(2k-1)\arccos e^{-\sigma t}]$. If the time expended by the computer on the execution of each of the indicated operations is denoted by t_1, t_2, t_3, and t_4, respectively, then the time expenditure to calculate a single value of $\bar{f}_n$ is:

$$t_0 = \left(\frac{n^2+3n}{2}+1\right)t_1 + (n^2 - n + 1)\,t_2 + (t_3 + t_4)\,n.$$

The proposed modification permits shortening this time to the minimum value $(t_1 + t_2 + t_3)n$.

Let us represent the solution of the system (10) in the general form

$$C_m = \sum_{i=1}^{m} A_{mi}\frac{4^i}{\pi}\sigma\bar{f}(p_i) = \frac{1}{\pi}\sum_{i=1}^{m} A_{mi}\frac{4^i}{2i-1}p_i\bar{f}(p_i), \quad m = 1, 2, \ldots, n,$$

where A_{mi} are elements of the matrix $\|a_{mi}\|^{-1}$. Substituting the coefficients into (9) and setting $\sigma = z/t$ (with z = const), we obtain

$$f_n(t) = \frac{1}{\pi}\sum_{m=1}^{n}\sin[(2m-1)\,\theta]\sum_{i=1}^{m} A_{mi}\frac{4^i}{2i-1}p_i\bar{f}(p_i) = \frac{1}{\pi}\sum_{m=1}^{n} p_m\bar{f}(p_m)\frac{4^m}{2m-1}\sum_{i=m}^{n} A_{im}\sin[(2i-1)\,\theta],$$

where $\theta = \arccos e^{-z}$ is a constant quantity. Also the coefficients

$$\delta_m^n(\theta) = \frac{1}{\pi}\frac{4^m}{2m-1}\sum_{i=m}^{n} A_{im}\sin[(2i-1)\,\theta],$$

whose values can be determined in advance and introduced into the computer in the form of a table, are constant when n and z are specified. Thus, a dependence to calculate an approximate value of the original $f_n(t)$ from a specified operator image $\bar{F}(p) = p\bar{f}(p)$ with the help of the simple formula

$$f_n(t) = \sum_{m=1}^{n}\delta_m^n(\theta)\,\bar{F}(p_m)$$

has been obtained.

The values of the coefficients $\delta_m^{16}(\pi/4)$ are tabulated in Table 1. The recommended values of the parameters n = 16 and z = (1/2) ln 2 are determined as a result of computational experiments with control examples. The numerical inversion of the image (7) was discussed as one of them. The accuracy of the approximation to the original (4) is illustrated in Fig. 2, in which the experimental points and the approximation are simultaneously shows by the filled circles, and the piecewise-parabolic approximation (4) is shown as a solid curve.

The values of the maximum ε_{1k} of the function (8) as well as the time intervals τ_k (with $k = 1, 2, \ldots l$) in which it is repeated are determined from the results of the numerical inversion of the image (6). Similar data ε_k^* and τ_k^* are obtained from the experiment (see Fig. 1). The values of ε_{1k} and τ_k can be treated as

TABLE 1

m	$\delta_m^{16}(\frac{\pi}{4})$	m	$\delta_m^{16}(\frac{\pi}{4})$
1	0	9	$0,540885164359385\cdot10^9$
2	$-0,960337403900965\cdot10^1$	10	$-0,118170226784452\cdot10^{10}$
3	$0,968020103132123\cdot10^3$	11	$0,184242612102173\cdot10^{10}$
4	$-0,338807036096242\cdot10^5$	12	$-0,203553615840318\cdot10^{10}$
5	$0,592489764710707\cdot10^6$	13	$0,155639872641486\cdot10^{10}$
6	$-0,606984127342018\cdot10^7$	14	$-0,783211919492181\cdot10^9$
7	$0,397567629286954\cdot10^8$	15	$0,233343137393532\cdot10^9$
8	$-0,175666085691838\cdot10^9$	16	$-0,311841059189513\cdot10^8$

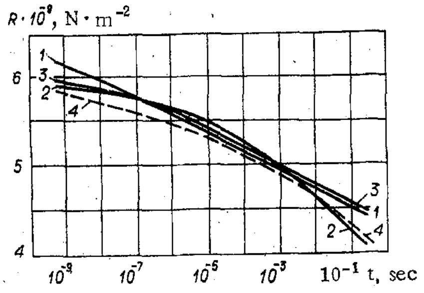

Fig. 3

functions dependent on the parameters $\alpha_1, \alpha_2, \ldots, \alpha_N$ of the relaxation kernel T from Eq. (3) as well as on the parameter $C_0 = \sqrt{E_0/\rho}$.

Using the least squares method, we shall construct the normalized mean-square deviation of the theoretical and experimental maximum points:

$$S(\alpha_1, \alpha_2, \ldots, \alpha_N, C_0) = \sum_{k=1}^{l} \left[\frac{\varepsilon_{ik} - \varepsilon_k^*}{\varepsilon_0}\right]^2 + \left[(\tau_k - \tau_k^*)\frac{C_0}{2L}\right]^2. \tag{11}$$

Thus calculation of the parameters of the relaxation kernel from the results of the wave experiment reduces to finding the minimum point of the function (11), which is dependent on N + 1 variables. This mathematical problem evidently has a unique solution but it does not pay to forget that the quantities ε_{ik}, ε_k^*, τ_k, τ_k^* are determined in practice with some relative errors whose values are denoted by β_ε, β_ε^*, β_τ, β_τ^*, respectively. It follows from this that when the condition

$$S(\alpha_1, \alpha_2, \ldots, \alpha_N, c_0) < [(\beta_\varepsilon + \beta_\tau^*)^2 + (\beta_\tau + \beta_\tau^*)^2]\, l \tag{12}$$

is satisfied, there is no sense in performing a further refinement of the parameters $\alpha_1, \alpha_2, \ldots, \alpha_N, C_0$, i.e., the inverse problem loses the uniqueness of a solution.

Condition (12) can serve as a criterion of whether or not a selected kernel is suitable for the description of dynamic processes with respect to a specific material. If with a given accuracy of the measurements and calculation the condition (12) is not satisfied at the minimum point of the function (11), one should proceed to consideration of a more complicated model.

The proposed method has been tested on examples of the calculation of the parameters of some viscoelastic models corresponding to polymethyl methacralate with a density of 1180 kg/m^3. The possibility of applying in this case the model of a rectangular spectrum [10] (model 1) with

$$T(t) = \frac{1-\alpha_1}{t \ln \frac{\alpha_3}{\alpha_2}}\left(e^{-\frac{t}{\alpha_3}} - e^{-\frac{t}{\alpha_2}}\right), \quad \overline{T}(p) = \frac{1-\alpha_1}{\ln \frac{\alpha_3}{\alpha_2}} \frac{\ln p + \frac{1}{\alpha_2}}{\ln p + \frac{1}{\alpha_3}};$$

Rzhanitsyn's kernel [20] (model 2) with

$$T(t) = \frac{1-\alpha_1}{t\Gamma(\alpha_3)}(\alpha_2 t)^{\alpha_3} e^{-\alpha_2 t}, \quad \overline{T}(p) = \frac{1-\alpha_1}{(1+\alpha_2 p)^{\alpha_3}}$$

and Rabotnov's kernel [17] (model 3) with

$$\overline{T}(p) = (1-\alpha_1)/(1+(\alpha_2 p)^{\alpha_3})$$

has been investigated. The values L = 0.985 m, τ = 25 μsec, $\tau_1^* = \tau_2^* = \tau_3^* = \tau_4^*$ = 950 μsec, $\varepsilon_1/\varepsilon_0 = 0.45$, $\varepsilon_2/\varepsilon_0 = 0.25$, $\varepsilon_3/\varepsilon_0 = 0.18$, and $\varepsilon_4/\varepsilon_0 = 0.15$ (see Fig. 1) were used as the initial data. As a result of the calculation for model 1 the parameter values $\alpha_2 = 10^{-9}$ sec and $\alpha_3 = 10^3$ sec were obtained; for model 2 – $\alpha_2 = 1$ sec and $\alpha_3 = 0.1$; and for model 3 – $\alpha_2 = 10^{-3}$ sec and $\alpha_3 = 0.15$. The values of the remaining parameters for all three models turned out to be practically identical: C_0 = 2300 m/sec and $\alpha_1 = 0.6$. The values of the function (11) at the minimum point also proved to be similar.

Plots 1-3 of the stress relaxation function

$$R(t) = \left(1 - \int_0^t T(y)\,dy\right)E_0, \tag{13}$$

plotted on the basis of the results obtained for models 1-3, respectively, and also from the data of [9], in which similar parameters of model 2 were determined from static experiments (curve 4), are given in Fig. 3. One can conclude from the arrangement of the curves that the values of the function (13) corresponding to the three relaxation kernels are in satisfactory agreement with each other for the parameter values indicated above and do not contradict the literature data. Moreover, calculations using model 2 show that the relaxation kernel parameters found by the proposed procedure provide more accurate agreement in the solution of a number of dynamic problems between the results of theory and experiment than do the analogous parameters given in [9].

One should note that the implementation of the method in question requires large expenditures both in the laboriousness of setting up the program for the computer and in machine time for the calculations (the calculation of four parameters of one model takes 15 sec on an ES-1050 computer). Notwithstanding this, there are definite advantages over the traditional methods of determining the kernel parameters.

The basic experiment of the proposed method is essentially of a nonsteady wave nature; during an analysis by the proposed procedure one can exclude the first burst which appears on the expanded picture of the strains followed by the first traverse of the length of the sample rod when the distortions due to three-dimensional effects, which occur in the impact region at the end, are still prominent enough, which increases the accuracy of the data obtained, for example, in comparison with [16]. One should also note that the experimental provision necessary for implementation of the method theoretically permits, by varying the length and thickness of the sample rods and the intensity and duration of the stress pulses introduced in them due to a decrease in the mass and flight velocity of the impacting objects, approaching most closely with the basic experiment for the loads which are involved in it those values for which it is necessary to do the calculations of specific structures subjected to impact–explosive actions. In addition the method, by encompassing the time range of appearance of the relaxation properties from approximately 200 μsec to 20 msec and even higher, which have been covered earlier by other methods with large errors or on the basis of analogies [7], permits finding along with the other nonwave methods, including static ones, the parameters of the kernels of the relaxation relations which most adequately describe the physicomechanical properties of rigid polymers by applying a complex analysis [21] of the results of a number of experiments differing in their nature by virtue of mutually supplementary data.

Thus, a new experimental method of determining the constants of Boltzmann–Volterra hereditary relationships based on a wave experiment with a rod has been proposed. The authors are grateful to V. N. Grivkov and V. I. Shlyakhov as well as to A. M. Kozlovtsev for useful discussions.

LITERATURE CITED

1. G. Kol'skii, "An experimental investigation of the mechanical behavior of linear viscoelastic media," Sb. Per. Mekh., No. 3 (1969).
2. T. Alfrey, The Mechanical Properties of High Polymers [Russian translation], IL, Moscow (1952).
3. F. Eyrich (ed.), Rheology, Theory and Applications [Russian translation], IL, Moscow (1962).
4. J. D. Ferry, Viscoelastic Properties of Polymers, Wiley (1970).
5. A. Tobol'skii, The Properties and Structure of Polymers [in Russian], Khimiya, Moscow (1964).
6. A. K. Malmeister, V. P. Tamuzh, and G. A. Teters, The Strength of Rigid Polymeric Materials [in Russian], Zinatie, Riga (1972).
7. P. M. Ogibalov, N. I. Malinin, V. P. Netrebko, and B. P. Kishkin, Construction Polymers (Methods of Experimental Investigation), Book 1, Moscow State Univ. (1972).
8. Jace W. Nunziato and Herbert J. Sutherland, "Acoustical determination of stress relaxation functions for polymers," J. Appl. Phys., 44, 184 (1973).
9. M. A. Koltunov, Creep and Relaxation [in Russian], Vysshaya Shkola, Moscow (1976).
10. S. M. Kokoshvili, V. P. Tamuzh, and Yu. S. Yanson, "The calculation of relaxation spectra from the results of dynamic tests," Mekh. Polim., No. 2 (1971).
11. W. Lethersich, "The rheological properties of dielectric polymers," Brit. J. Appl. Phys., 1, No. 11 (1950).
12. J. J. Benbow, "The dynamic mechanical properties of some organic glasses," Proc. Phys. Soc., Sec. B, 67, Pt. 2 (1954).
13. H. Blumenhauer (ed.), A Handbook on Materials Testing [Russian translation], Metallurgiya, Moscow (1979).
14. I. V. Zashchuk, Electronics and Acoustical Methods for the Testing of Structural Materials [in Russian], Vysshaya Shkola, Moscow (1968).
15. K. W. Hillier, "A vibrating cantilever method for the investigation of the dynamic elasticity of high polymers," Proc. Phys. Soc., Sec. B, 64, Pt. 11 (1951).
16. V. P. Muzychenko, "Stress waves in structural elements," VINITI, Dep. No. 1775-79, Daugavpils (1979).
17. Yu. N. Rabotnov, Creep Problems of Structural Members, North-Holland (1969).
18. V. P. Muzychenko and N. M. Ivanov, "A computational algorithm of a polymeric product subjected to a transient action of specified form," in: Nonsteady Loading of Polymeric Composite Structures [in Russian], No. 2, Izd. VVAIU, Riga (1972).
19. A. Papoulis, "A new method of inversion of the Laplace transform," Q. Appl. Mat., 14, No. 4 (1957).
20. A. R. Rzhanitsyn, Some Problems of the Mechanics of Systems Which Deform in Time [in Russian], Gos. Izd. Tekhn.-Teor. Lit., Moscow–Leningrad (1949).

21. A. M. Kozlovtsev, V. P. Muzychenko, and A. M. Bragov, "A theoretical–experimental method for the determination of the viscoelastic characteristics of polymeric composite materials," in: Abstracts of Lectures of the 5th All-Union Conference on Composite Materials [in Russian], No. 2, Moscow State Univ. (1981).

DYNAMICS OF THE DEFORMATION OF A SPHERICAL PORE IN A PLASTIC MATERIAL

A. V. Attetkov, V. V. Selivanov, and V. S. Solov'ev

UDC 662.215.5+539.374.1

Localized high-temperature regions ("hot spots") exert a decisive effect on the stimulation of chemical decomposition in high-density heterogeneous explosives. It follows from the general laws of the dynamics of deformation of porous media that their deformation characteristics are determined by the competition of two distortion mechanisms: the production of cracks (or slip bands), and pore deformation (collapse) [1]. Consequently, the formation of localized hot spots during shock triggering of heterogeneous explosives may involve pore deformation, shear fracture, and the slip ratio of the particles of the explosive. The present article is devoted to a consideration of the general laws of pore deformation in a plastic material, and the heating of its surface during its collapse. The fundamental equation of a porous material which is being compressed was proposed in [2]. A porous material was regarded as an ideal homogeneous continuum with an additional kinematic variable α, defined as the ratio of the specific volume of the porous material to the specific volume of the matrix material. A similar approach was used in [3] in investigating the deformation of granular materials. This permits the treatment of the deformation of a porous material within the framework of a continuous medium model. Theoretical papers [4, 5] have been devoted to a description of the behavior of the parameter α. The problem reduces to the consideration of the collapse of a hollow sphere under the action of pressure applied to the outer surface, where the ratio of the inside and outside radii determines the porosity of the given material. The results obtained showed that the compressibility of the material and the shear modulus G determining the elastic and elastic–plastic phases of the pore collapse do not have a significant effect. A change of porosity occurs when all the material goes over into a state of plastic flow. Thus, for load intensities an order of magnitude larger than the yield strength Y of the material, it is accurate enough to employ a rigid-plastic model of the medium.

Let us consider the deformation of a pore in a plastic material under the action of a pressure p uniformly distributed over the outer surface. Let a_0 and a be, respectively, the initial and present values of the pore radius, b_0 and b the initial and present values of the radius of the sphere. We characterize the initial porosity m_0 of the material by the ratio of the specific volume of the pores to the volume of the continuous material. Then

$$m_0 = (a_0/b_0)^3. \tag{1}$$

We assume that the material is homogeneous, isotropic, and incompressible, with a density ρ, and that it satisfies the Mises–Hencky or the Tresca–St. Venant yield condition with a constant yield strength Y.

Using the assumption of the incompressibility of the material (ρ = const), we determine the integral of the equation of continuity

$$\partial\rho/\partial t + \partial(\rho v)/\partial r + 2\rho v/r = 0$$

in the form

$$v = \dot{a}(a/r)^2, \tag{2}$$

where r is an Eulerian coordinate; v, radial velocity; and $\dot{a} = da/dt$, velocity of the pore boundary.

Substituting the last expression and the derivatives $\partial v/\partial t$ and $\partial v/\partial r$ into Euler's equation for the case of central symmetry

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 127-131, January-February, 1983. Original article submitted December 23, 1981.

0021-8944/83/2401-0111$07.50